Cluster-based network model for drag reduction mechanisms of an actuated turbulent boundary layer

Article *in* PAMM · November 2019

DOI: 10.1002/pamm.201900219

Richard Semaan
Technische Universität Braunschweig

Marian Albers
RWTH Aachen University

Pascal Sylvain Meysonnat
RWTH Aachen University

Rishabh Ishar
Helmholtz-Zentrum Dresden-Rossendorf

Cluster-based network model for drag reduction mechanisms of an actuated turbulent boundary layer

Daniel Fernex [1,*], **Richard Semaan** [1], **Marian Albers** [2], **Pascal S. Meysonnat** [2], **Wolfgang Schröder** [2], **Rishabh Ishar** [3], **Eurika Kaiser** [4], and **Bernd R. Noack** [5]

[1] Institute of Fluid Mechanics, Technische Universität Braunschweig, Germany
[2] Institute of Aerodynamics, RWTH Aachen University, Germany
[3] Department of Mechanical Engineering, Massachusetts Institute of Technology, USA
[4] University of Washington, Department of Mechanical Engineering, USA
[5] Laboratoire d'Informatique pour la Mécanique et les Sciences de l'Ingénieur (LIMSI-CNRS), Orsay, France

We introduce a novel data-driven reduced-order modeling approach, a *Cluster-Based Network Model (CBNM)*. Starting point is a set of time-resolved snapshots associated with one or multiple control laws. These snapshots are coarse-grained into dozens of centroids using k-means++ clustering. The dynamics is modelled in a network between these centroids comprising the transition probability and corresponding transit time. The transition parameters depend on the control law. CBNM is successfully applied to an actuated turbulent boundary layer flow. The results show that CBNM is an attractive alternative to POD models as the model is human interpretable and dynamically robust by construction.

1 Introduction

Reduced-Order Models (ROM) of fluid flows aim to capture the dominant coherent structures of high-fidelity models—trading accuracy in a high-dimensional state space with simplicity in a low-dimensional, ideally human-interpretable one. ROM are key enablers for physical understanding, computationally affordable optimization, and model-based control [1]. Aubry et al. [2] have pioneered POD models for a turbulent boundary layer. However, the construction of POD models are challenging for broadband turbulence, multiple operating conditions, and slowly changing coherent structures.

In this work, we propose a Cluster-Based Network Model (CBNM), a simple and dynamically robust alternative to POD models. CBNM extends CROM [3], i.e. cluster-based Markov models, based on time-resolved, statistically representative flow snapshots. The model coarse-grains the snapshots into centroids and describes the temporal dynamics via a probabilistic network of transitions derived from the data. The model is generally more accurate than CROM and allows to integrate multiple operating conditions. CBNM is applied to a drag reduction study of turbulent boundary layer subject to dozens of transverse surface waves [4].

2 Methodology

The cluster-based network model methodology is presented in Fig. 1. First, the time-resolved snapshots from numerical simulations or experimental data are collected. Then, the snapshots are grouped into clusters, C_k, $k = 1, \dots, K$, represented by their respective centroids $\boldsymbol{c}_k$ using the k-means++ algorithm [3]. The *Direct Transition Matrix (DTM)* $\boldsymbol{Q}$ describes the

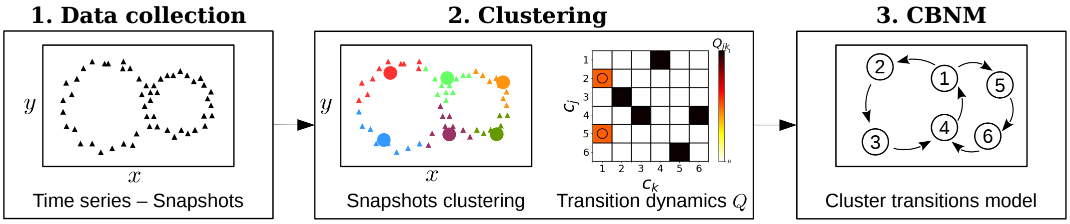

Fig. 1: Schematic of the pipeline for the cluster-based network model.

transition probability to move from cluster C_k to cluster C_j: $Q_{jk} = n_{jk}/n_k$ if $k \neq j$, and 0 otherwise. n_k is the number of snapshots in C_k and n_{jk} the number of snapshots moving directly from C_k to C_j. The diagonal terms of $\boldsymbol{Q}$ are zero unless the trajectory is trapped in the cluster and cannot leave. The sum of each column is one. C_i is the chosen initial 'zeroth' cluster. The mth cluster is randomly picked based on the $(m-1)$th cluster as starting point and on the transition probability given by $\boldsymbol{Q}$.

* Corresponding author: e-mail d.fernex@tu-braunschweig.de, phone +49 531 391-94266

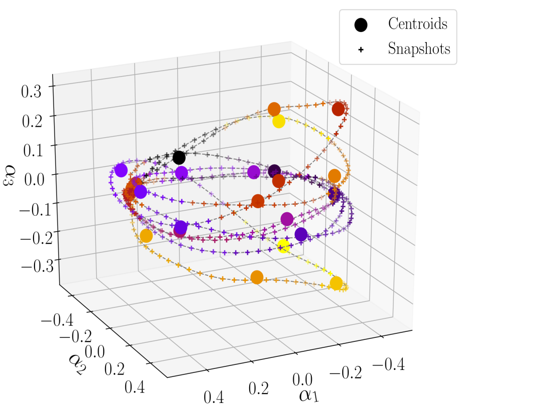

Fig. 2: Proximity map in the space of the first three POD modes α_1, α_2 and α_3 associated with the centroids. The snapshot coloring is based on their cluster affiliation.

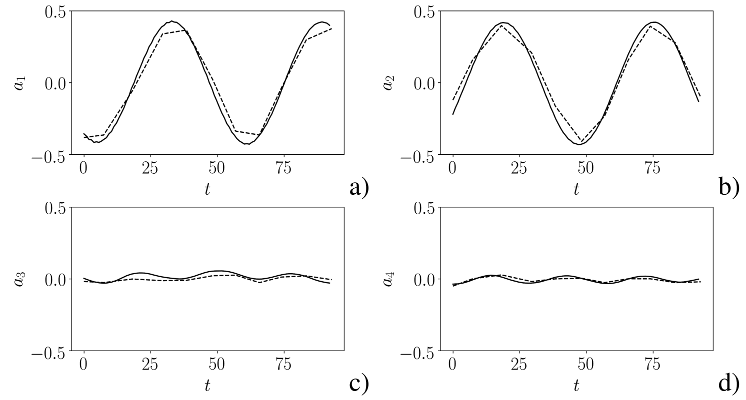

Fig. 3: POD mode coefficients for the first four modes (figures a, b, c and d, respectively) from the data (continuous lines) and from the CBNM (dashed lines).

The transition time τ_{jk} from cluster C_k to C_j is the mean value of the numerically observed transitions times. The velocity field $\boldsymbol{u}(\boldsymbol{x}, t)$ is assumed to move uniformly and linearly from centroid $\boldsymbol{c}_k$ to $\boldsymbol{c}_j$. In other words, the velocity reads $\boldsymbol{u}(\boldsymbol{x}, t) = \alpha_{jk}(t)\boldsymbol{c}_j(\boldsymbol{x}) + (1 - \alpha_{jk}(t))\boldsymbol{c}_k(\boldsymbol{x})$ with the transition parameter $\alpha_{jk} = (t - t_m)/\tau_{jk}$, where t_m is the 'escape' time from the mth visited cluster C_k.

3 Results

The CBNM is applied to Large-Eddy-Simulation (LES) data of a turbulent boundary layer undergoing spanwise transversal surface waves which result in a skin friction change [4]. Details about the setup can be found in [5]. The operating conditions are defined by the wavelength λ^+, the period T^+, and the amplitude A^+ in inner units. The CBNM is implemented on a case with 420 snapshots, and defined by the actuation parameters $\lambda^+ = 1000$, $T^+ = 120$ and $A^+ = 40$. Twenty-four clusters are chosen. To reduce computational cost, clustering is performed on lossless POD mode coefficients of the data instead of the snapshots. This leads to the same results [3]. Fig. 2 depicts the snapshots and the centroids in a proximity map in the space of the first three POD modes associated with the centroids (see [3] for details about the visualization). The dashed line shows the snapshot-based trajectory. The snapshots are colored according to their cluster affiliation. Fig. 3 shows the first four POD mode coefficients from the data compared to those obtained by the CBNM. The good agreement between the two curves demonstrates the accuracy of the model. The wave actuation effect is clearly present in the first two modes. The amplitudes of the next coefficients decreases rapidly.

4 Conclusion and outlook

The presented cluster-based network model (CBNM) robustly resolves the coherent structure dynamics of broadband turbulence. CBNM is dynamically more accurate than CROM as the deterministic transition times are resolved. With its robust dynamics that cannot leave the domain of the data, CBNM is an attractive alternative to POD models. This robustness comes at the price of the loss of explorative power from 'new' superpositions of POD models.

CBNM is successfully applied on an actuated turbulent boundary layer flow, where it accurately reproduces the flow dynamics. CBNM can be improved in numerous ways. One example includes more realistic transitions with a non-linear trajectory and non-uniform velocity between the centroids. Furthermore, CBNM can also model flows with multiple operating conditions.

Acknowledgements The research was funded by the Deutsche Forschungsgemeinschaft (DFG) in the framework of the research project SE 2504/2-1.

References

[1] S. L. Brunton and B. R. Noack, Applied Mechanics Reviews **67**(5), 050801–050801–48 (2015).
[2] N. Aubry, P. Holmes, J. L. Lumley, and E. Stone, Journal of Fluid Mechanics **192**(July), 115–173 (1988).
[3] E. Kaiser, B. R. Noack, L. Cordier, A. Spohn, M. Segond, M. Abel, G. Daviller, J. Östh, S. Krajnović, and R. K. Niven, Journal of Fluid Mechanics **754**(September), 365–414 (2014).
[4] M. Albers, P. S. Meysonnat, W. Schröder, R. Semaan, and B. R. Noack, Proc. Appl. Math. Mech. (2019), submitted.
[5] R. Ishar, E. Kaiser, M. Morzynski, D. Fernex, R. Semaan, M. Albers, P. S. Meysonnat, W. Schröder, and B. R. Noack, J. Fluid Mech. (2019), (in print).